SOLUTIONS FOR QUESTIONS ON RADIATIVE HEAT TRANSFER – 2021-01-11:

7 a)

The plate only sees the duct:

$$F_{11} = 0$$

$$F_{12} = 1$$

Using the reciprocity relation:

$$F_{21} = F_{12}\frac{A_1}{A_2} = F_{12}\frac{2R_2L}{\pi 2R_2L} = F_{12}\frac{1}{\pi} = \frac{1}{\pi} \approx 0.318$$

$$F_{12} = 1 - F_{21} = 0.682$$

7 b)

The sphere is a convex body and can't see itself. Since we have an infinite plane, half of the radiation from the sphere must reach the plane at some point:

$$F_{11} = 0$$

$$F_{12} = 0.5$$

$$F_{1\infty} = 1 - F_{11} - F_{12} = 0.5$$

The small spehere is much smaller than the infinite plane, meaning that the area fraction is infinitely small:

$$F_{21} = F_{12}\frac{A_1}{A_2} = \{A_2 \gg A_1\} = 0$$

The flat surface can't see itself:

$$F_{22} = 0$$

$$F_{2\infty} = 1$$

8 a) Assume that the reflectivity is independent on the solar direction. The total reflectivity can then be expressed as:

$$\rho = \frac{\int_0^\infty \rho_\lambda G_\lambda(T) d\lambda}{\int_0^\infty G_\lambda\,(T) d\lambda}$$

Given that the solar radiation is blackbody radiation we can calculate the total reflectivity as:

$$\rho = \rho_1 \frac{\int_0^{\lambda_1} E_{\lambda b}(\lambda, T)\, d\lambda}{E_b(T)} + \rho_2 \frac{\int_{\lambda_1}^{\lambda_2} E_{\lambda b}(\lambda, T)\, d\lambda}{E_b(T)} + \rho_3 \frac{\int_{\lambda_2}^{\lambda_3} E_{\lambda b}(\lambda, T)\, d\lambda}{E_b(T)} + \rho_4 \frac{\int_{\lambda_3}^{\lambda_\infty} E_{\lambda b}(\lambda, T)\, d\lambda}{E_b(T)}$$

$$\rho = 0 + 0.90 \frac{\int_{0.2}^{1} E_{\lambda b}(\lambda, T)\, d\lambda}{E_b(T)} + 0.50 \frac{\int_{1}^{2} E_{\lambda b}(\lambda, T)\, d\lambda}{E_b(T)} + 0$$

$$\rho = 0.90 * F_{0.2\to 1} + 0.50 * F_{1\to 2}$$

From table 12.2 in the course book, the portion of the blackbody radiation corresponding to each interval can be found. First, from $\lambda = 0$, to each wavelength:

$$F_{0\to\lambda} = \frac{\int_0^{\lambda} E_{\lambda b}(\lambda, T)\, d\lambda}{E_b(T)}$$

$$\left.\begin{array}{lll} \lambda_1 T = 0{,}2 * 5800 = 1160\ \mu mK & \to & F_{0\to\lambda_1} = 0.0018 \\ \lambda_2 T = 1 * 5800 = 5800\ \mu mK & \to & F_{0\to\lambda_2} = 0.7202 \\ \lambda_3 T = 2 * 5800 = 11600\ \mu mK & \to & F_{0\to\lambda_3} = 0.9410 \end{array}\right\}$$

We can now calculate the portion of the blackbody radiation that corresponds to each wavelength interval:

$$F_{\lambda_1\to\lambda_2} = F_{0\to\lambda_2} - F_{0\to\lambda_1} = 0.7184$$

$$F_{\lambda_2\to\lambda_3} = F_{0\to\lambda_3} - F_{0\to\lambda_2} = 0.2208$$

The total reflectivity becomes:

$$\rho = 0.90 * 0.7184 + 0.50 * 0.2208 = 0.75696 \approx 0.76$$

8 b) The concentration factor is given as:

$$C = \frac{A_{mirrors}}{A_{absorber}} = \frac{2500 * 1 * 1}{5} = 500$$

8 c) All solar radiation is assumed to be reflected towards the absorber, the reflectivity we calculated above tells us how much solar radiation that is directed towards the absorber.

We neglect the emission from the mirrors. We assume that the absorber is grey $(\alpha = \varepsilon)$.

The absorber efficiency can be expressed as:

$$\eta = \frac{Q_{medium}}{Q_{in}} = \frac{\alpha Q_{in} - Q_{loss}}{Q_{in}} = \alpha - \frac{Q_{loss}}{Q_{in}}$$

Where the incident heat flux to the absorber surface can be calculated as:

$$Q_{in} = A_{mirror} \rho G_{dir}$$

And the heat loss for the absorber (grey assumption):

$$Q_{loss} = A_{absorber} \varepsilon \sigma T_{medium}^4$$

Gives the absorber efficiency as:

$$\eta = \alpha - \frac{\alpha \sigma T_{medium}^4}{A_{mirror} \rho G_{dir}} = 0.90 - \frac{5 * 0.90 * 5.67 * 10^{-8} * (800 + 273)^4}{2500 * 0.76 * 950} = 0.7126 \approx 0.71$$